Inverse Trigonometric Functions:

Key Points:

- An inverse function is one that "undoes" another function. The domain of an inverse function is the range of the original function and the range of an inverse function is the domain of the original function.

Trig Functions	Inverse Trig Functions
Domain: Measure of an angle	Domain: Ratio
Range: Ratio	Range: Measure of an angle

- Because the trigonometric functions are not one-to-one on their natural domains, inverse trigonometric functions are defined for restricted domains.

- The **inverse sine function** $y = \sin^{-1} x$ means $x = \sin y$. The inverse sine function is sometimes called the **arcsine** function and denoted as $arc\sin x$

$$y = \sin^{-1} x \text{ has domain } [-1,1] \text{ and range } \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

- The **inverse cosine function** $y = \cos^{-1} x$ means $x = \cos y$. The inverse sine function is sometimes called the **arccosine** function and denoted as $arc\cos x$

$$y = \cos^{-1} x \text{ has domain } [-1,1] \text{ and range } [0, \pi]$$

- The **inverse tangent function** $y = \tan^{-1} x$ means $x = \tan y$. The inverse tangent function is sometimes called the **arctangent** function and denoted as $arc\tan x$

$$y = \tan^{-1} x \text{ has domain } (-\infty, \infty) \text{ and range } \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$$

- Special angles are the outputs of inverse trigonometric functions for special input values; for example,

$$\frac{\pi}{4} = \tan^{-1}(1) \ , \frac{\pi}{6} = \sin^{-1}\left(\frac{1}{2}\right)$$

- A calculator will return an angle within the restricted domain of the original trigonometric function.

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Compositions of a trigonometric function and its inverse

$$\sin(\sin^{-1} x) = x \text{ for } -1 \le x \le 1$$
$$\cos(\cos^{-1} x) = x \text{ for } -1 \le x \le 1$$
$$\tan(\tan^{-1} x) = x \text{ for } -\infty < x < \infty$$

$$\sin^{-1}(\sin\ x) = x \text{ only for } -\frac{\pi}{2} \le x \le \frac{\pi}{2}$$
$$\cos^{-1}(\cos\ x) = x \text{ only for } 0 \le x \le \pi$$
$$\tan^{-1}(\tan\ x) = x \text{ only for } -\frac{\pi}{2} < x < \frac{\pi}{2}$$

Inverse Trigonometric Functions Videos:

- Writing a Relation for an Inverse Function: Example 1
- Evaluating Inverse Trigonometric Functions for Special Input Values: Examples 2-5
- Using Calculator to Evaluate Inverse Trigonometric Functions: Example 6
- Applying Inverse Trigonometric Function to Right Triangle : Example 7
- Evaluating Composition of Trigonometric Functions and its inverse: Examples 8-12
- Evaluating Composition of Trigonometric Functions and the inverse of another Trigonometric Function: Examples 13-14
- Evaluating Composition of Trigonometric Functions and the inverse of another Trigonometric Function: Examples 15-16
- Evaluating Composition of Trigonometric Functions and the inverse of another Trigonometric Function: Example 17

Practice Exercises:

For the following exercises, evaluate the expressions.

1. $\sin^{-1}\left(-\frac{1}{2}\right)$
2. $\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right)$
3. $\tan^{-1}\left(-\sqrt{3}\right)$
4. $\tan^{-1}\left(\sqrt{3}\right)$

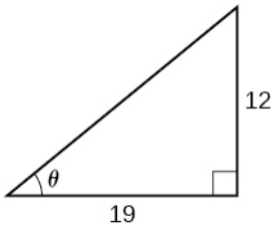
For the following exercises, use the calculator to evaluate each expression. Express answeres to the nearest hundredth.

5. $\cos^{-1}(-0.4)$

6. $\arccos\left(\frac{3}{5}\right)$

7. $\tan^{-1}(6)$

8. Find the angle θ in the given right triangle. Round answer to the nearest hundredth.



For the following exercises find the exact values.

9. $\tan^{-1}(\sin(\pi))$

10. $\tan^{-1}\left(\sin\left(\frac{\pi}{3}\right)\right)$

11. $\tan^{-1}\left(\sin\left(\frac{4\pi}{3}\right)\right)$

12. $\tan^{-1}\left(\sin\left(-\frac{5\pi}{2}\right)\right)$

13. $\sin\left(\cos^{-1}\left(\frac{3}{5}\right)\right)$

14. $\cos\left(\tan^{-1}\left(\frac{12}{5}\right)\right)$

For the following exercises, find the exact value of the expression in terms of x.

15. $\cos\left(\sin^{-1}\left(\frac{1}{x}\right)\right)$

16. $\cos\left(\sin^{-1}\left(\frac{x}{x+1}\right)\right)$

Answers:

1. $-\frac{\pi}{6}$

2. $\frac{3\pi}{4}$

3. $-\frac{\pi}{3}$

4. $\boldsymbol{\frac{\pi}{3}}$

5. 1.98

6. 0.93

7. 1.41

8. 0.56

9. 0

10. $\frac{\pi}{3}$

11. $-\frac{\pi}{3}$

12. $-\frac{\pi}{4}$

13. $\frac{4}{5}$

14. $\frac{5}{13}$

15. $\frac{\sqrt{x^2-1}}{x}$

16. $\frac{\sqrt{2x+1}}{x+1}$